Assigned Number for this Exam ________ **(DO NOT use your name)**

Solid Mechanics PhD Qualifying Exam
January 2004

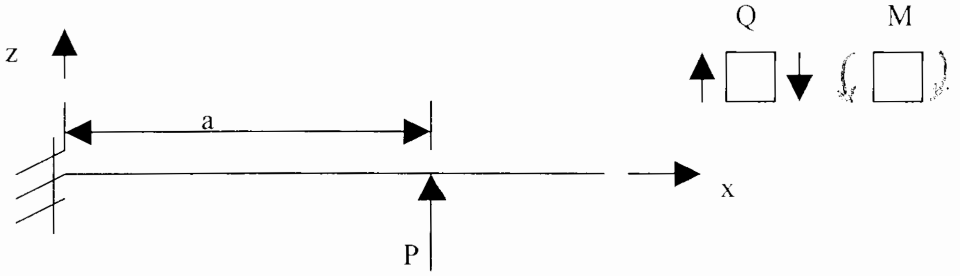
Closed Book---2 hours

You are required to solve four problems from the following five. Circle below the four problems you have chosen. (The other problem will not be graded or counted.) Each problem is worth 25 points.

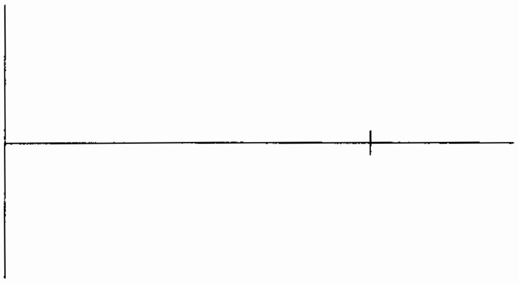
1 2 3 4 5

Use your time wisely. Check the point values within the problems and plan your time accordingly.

1) Consider the use of the elementary beam Euler-Bernoulli beam theory for the laminated beam of length "L" shown below.



Calculate and draw the shear and moment diagrams. **Signs count;** label the important points.
Hint: suggest you solve for the reactions first.
(2@4 points)



b) For the beam of part (a), give the four matching conditions (m.c.) at x=a **in terms of the displacements w_1 and w_2 (and their derivatives).**

Hint: $M= -(E^bI)w''$, $Q= -(E^bI)w'''$; consider equilibrium of the "joint" (x=a)
(4 + 6 points)
Further question: Give the shear and moment m.c. in terms of w_1 and w_2 if the beam suddenly changes its cross-section from $(E^bI)_1$ to $(E^bI)_2$ at x=a. (4+3 pts)

2) Consider a spherical inclusion of material "1" with a radius "R" in a "very large" surrounding medium of material "2". The entire space is initially without stress at some temperature and then the temperature is changed uniformly by a value T. The only independent variable is the radius measured from the center of the inclusion "r", but the normal three variables for this coordinate system are (r, α, θ). See figure.

a) Discuss the use of equilibrium, constitutive, and compatibility equations in each domain. (9 pts)

b) What kinds of matching conditions would you apply at the interface? Are any components of the field variables in the m.c. trivial? (9 pts)

c) What kinds of conditions would you apply as the value of r→infinity? (7 pts)

The surfaces r = const and θ = const are circular cylinders and radial planes as in case a above, while the surfaces z = const are planes parallel

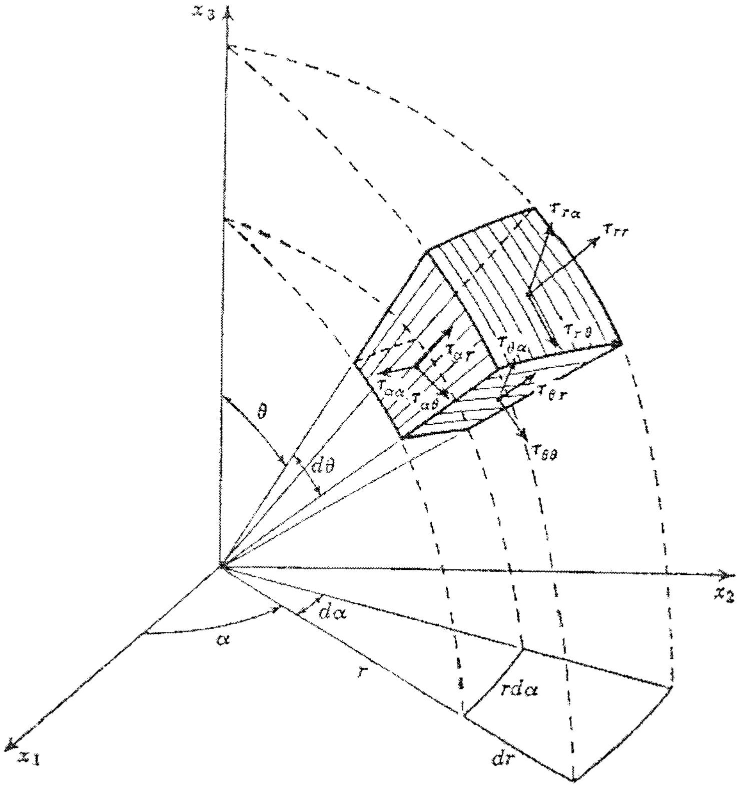
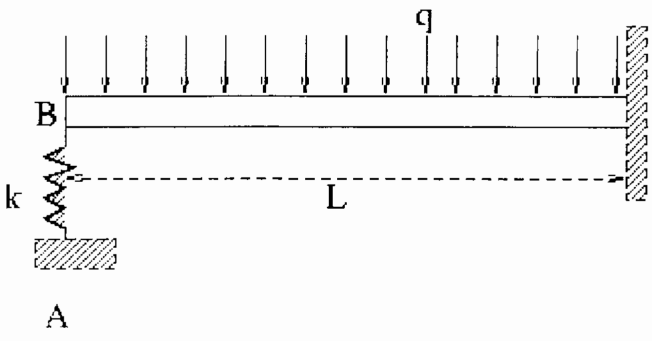


Fig. 38

to the xy-plane (Fig. 37). Substituting the values of the metric coeffi-

3 The left end of the cantilever beam in the figure below is supported at B by a linear spring of stiffness k, which is unstressed when uniformly distributed load q is absent. Determine the force in the spring and deflection v_B at B due to application of the load q. (25 pts)



4. (a) Given that $A_{ij}X_iX_j$ is a scalar for arbitrary vector X_i. Is A_{ij} a tensor? Prove your answer. (10 pts)

(b) Recall Green's first identity:

$$\int_V \phi \nabla^2 \psi = \int_S \phi \frac{\partial \psi}{\partial n} - \int_V (\nabla \phi) \cdot (\nabla \psi).$$

Given $\nabla^2 u = \nabla^2 w$ in volume V and $u = w$ on its surface S, show that $u = w$ in V. (15 pts)

5. A thick prismatic bar of rectangular cross section a x b and length h is stood on end with gravity the only load. The material is linearly elastic, with Young's modulus E and Poisson's ratio ν, and uniform (mass) density ρ.

(a) Describe St. Venant's principle. (3 pts)

(b) Find the stresses and displacements away from the bottom. (15 pts)

(c) What are appropriate support conditions so that the solution is valid everywhere and boundary conditions are satisfied on all surfaces? (7 pts)

Use a right hand coordinate system with the origin at the center of the bottom surface and the z axis directed upward.