

Department of Mechanical and Aerospace Engineering
 2003 Ph.D Qualifying Exam
 Jan 10, 2003
Dynamics and Control

Assigned # _____

NO TEXTBOOKS OR NOTES ALLOWED. YOU ARE REQUIRED TO ANSWER 4 OF THE 5 QUESTIONS.

Problem A:

You are given a slider-crank mechanism below (whose position-level configuration has been previously calculated and tabulated).

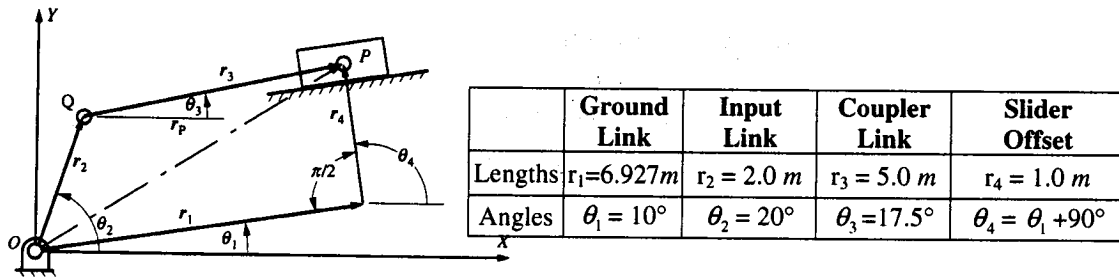
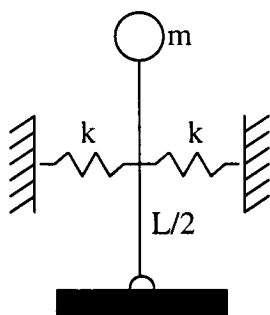


Figure 3: Configuration & Nomenclature of a Slider-Crank Mechanism

Now given that the input crank is rotated at an angular velocity ($\dot{\theta}_2 = 1 \text{ rad/sec}$), compute the velocities of all the links in the system ($\dot{r}_i, \dot{\theta}_i$).

Problem B:

Consider the inverted pendulum connected by two equal springs shown below:
 Assume that the springs are undeflected when in the vertical position and that the mass m of the ball at the end of the pendulum rod is substantially larger than the mass of the rod. If the rod is of length L and the springs are attached at the point $L/2$, determine the equations of motion, linearize the model and determine the natural frequency and determine the relationship between the system parameters which correspond to the initiation of unstable motion.



Problem C:

Consider the following state matrices, where α and β are some real-valued constants:

$$A = \begin{bmatrix} 1 & \alpha & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix}, \quad C = \begin{bmatrix} 0 & \alpha & 0 \\ 0 & \beta & 1 \end{bmatrix}$$

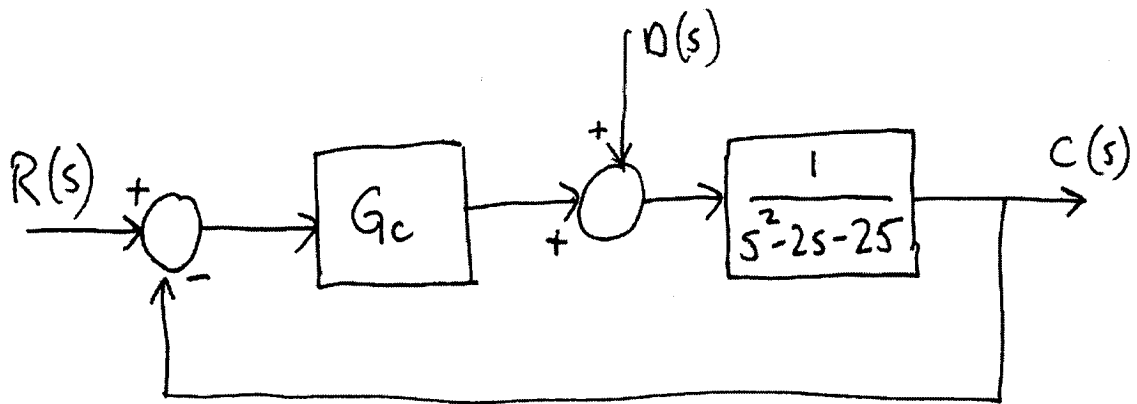
- a) For what values of α and β is the system controllable?
- b) For what values of α and β is the system observable?
- c) Determine the eigenvalues of the system?
- d) Determine the similarity transformation to transform the system to the Jordan Canonical form. Verify your result.

Problem D:

1) For the system shown, design a controller (represented by the transfer function, G_c) using one or more of the elements P, I, and/or D, such that

- a) The system is stable
- b) The time constant is less than 0.5
- c) The steady-state error in response to a step input is zero
- d) The steady-state response to a step disturbance is zero

2) After finding your design in part (1), sketch the magnitude and phase Bode plots for the closed-loop primary transfer function.



Problem E:

For the following system, determine each of the isolated equilibrium points and determine the stability of each of them:

$$\dot{x}_1 = -x_1 + x_2(1 + x_1)$$

$$\dot{x}_2 = -x_1(1 + x_1)$$