MECHANICS

Closed Book
Time : 2 hours

January 7, 2003

Student Number for Qualifying exam: ______________________
(Please do not use your name)
You are required to solve any 4 problems. Each is worth 25 %.
Circle below the four problems you have solved.
1. 2. 3. 4. 5.

1. The following equations apply to a linear elastic body equilibrium

$$\epsilon_{ii} = \frac{1}{2}(u_{i,j} + u_{j,i})$$
$$e_{pir}e_{qjs}\epsilon_{ij,rs} = 0$$
$$\tau_{ij,j} + B_i = 0$$
$$\epsilon_{ij} = \frac{1+\nu}{E}\tau_{ij} - \frac{nu}{E}\tau_{kk}\delta_{ij}$$
$$T_i = \tau_{ij}n_j$$
$$\int_V (\ )_{,i}\, dV = \int_S (\ )n_i ds$$

Show that

$$\int_S T_i u_i dS + \int_V B_i u_i dV = \int_V \tau_{ij}\epsilon_{ij} dV$$

2. a) Consider the simply supported prismatic cantilever column shown in figure 1. Compute the critical buckling load using the following differential equation (sum of moments at x):

$$EI\frac{\partial^2 v}{\partial x^2} + Pv = Pe$$

b) Use the Rayleigh-Ritz method with $v(x) = e(\frac{x}{l})^2$ to approximate the buckling load. Please use

$$M = P(e - v)$$
$$\delta V = \delta \int_0^l \frac{M^2}{2EI} dx$$
$$\delta W = P\delta \int_0^l \frac{1}{2} \frac{dv}{dx}^2 dx$$

What is the error in the approximate buckling load ?

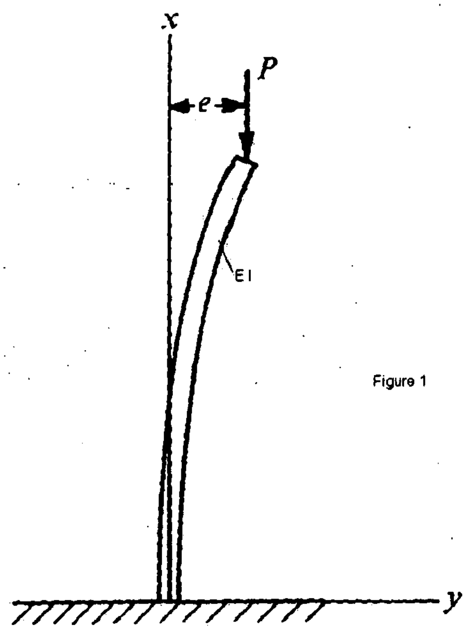

Figure 1

3. A thin walled tube has the dimensions of R=12 in and t=0.1 in. The yield point in tension in $\sigma_0 = 50,000psi$. An axial weight P=200,000 lb is hung as shown in figure 2. For what value of M will the tube yield, according to the yield criteria

$$(\sigma_i - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_0^2 \quad (1)$$

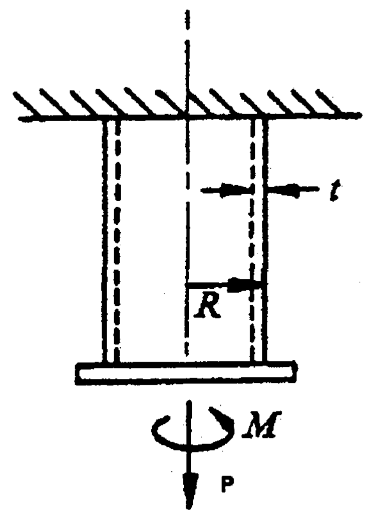

Figure 2

4. The thin cantilever beam shown in figure 3 is subjected to a uniform shearing stress τ_0 along the upper surface, y=+D; while the surfaces y=-D and x=L are stress free. Determine if the Airy stress function

$$\phi = \frac{1}{2}\tau_0(xy - \frac{xy^2}{D} - \frac{xy^3}{D^2} + \frac{Ly^2}{D} + \frac{Ly^3}{D^2} \tag{2}$$

satisfies the boundary conditions for this problem.

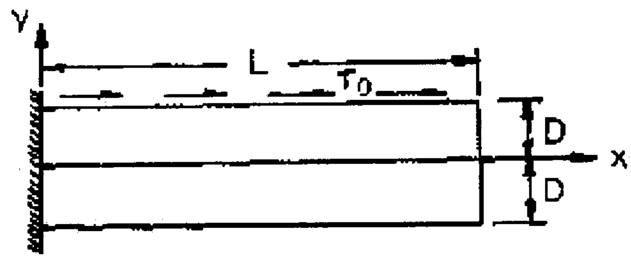

Fig. 5

5. A prismatic beam of rectangular cross section is oriented with its top face horizontal. Let the x axis be through the centroids of cross sections, the y axis vertical and the z axis horizontal, xyz being rectangular Cartesian. The beam is subjected to "pure bending", i.e., loaded only by couple vectors at the ends, parallel to the z axis.

   a) By considering two identical adjacent elements of the beam bounded by planes perpendicular to the x axis when unloaded, demonstrate that, away from the ends, cross sections plane prior to loading remain plane after loading, and that normal stresses and strains in the x direction are linear in y. Reminder: there are no axial end loads.

   b) Assume that the axial stress τ_{xx} = Ky, K a constant, other $\tau_{ij} = 0$. Show that this stress distribution satisfies the equilibrium equations and the compatibility equations of linear elasticity. Determine the displacement field in terms of several constants of integration and K. It is not necessary to evaluate these constants (or K).