QUALIFYING EXAM
MATHEMATICS
JAN. 10, 2003

1 (10%).

a (5%) Let $\{e_1,e_2\}$ be the standard basis in $\mathbf{R}^2$ find the matrix **A** for the transformation (reflection):

$$A(x_1,x_2) = (x_1,-x_2)$$

then how that **A** is a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$.

b (5%) Find the vectors in $\mathbf{R}^2$ such that their directions, but not magnitudes remain unchanged under this transformation.

2 (50%) Consider the boundary value problem:

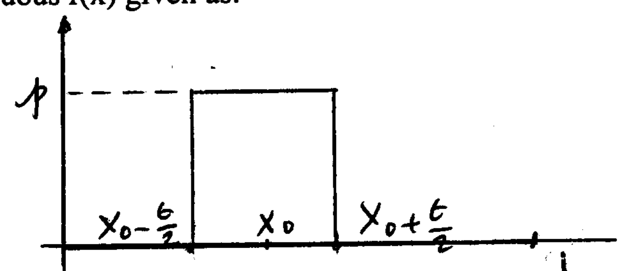
$$-T\frac{d^2u}{dx^2} = f(x), 0<x<1; u(0) = u(1) = 0$$

where T is a constant and f(x) is a given continuous function. This is a model for the deflection u(x) of a taut, flexible string of unit length with fixed ends under the load of f(x) force/(unit length).

a (15%) Solve for u(x) for an arbitrary f(x).

b (15%) Solve for u(x) for a piecewise continuous f(x) given as:

$$f(x) = \begin{cases} 0, & 0 \le x < x_0 - \frac{\varepsilon}{2}, \\ p, & |x - x_0| \le \frac{\varepsilon}{2}, \\ 0, & x_0 + \frac{\varepsilon}{2} < x < 1. \end{cases}$$



c (20%) Let $\varepsilon \to 0$ but keeping $p\varepsilon = 1$, calculate the limit of the solution u(x) in part b.

3. (40%) Consider the heat conduction problem in a bar of length π:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + e^{-t}\sin x, \; u(0,t) = 0, \; u(\pi,t) = 1, \; u(x,0) = x(\pi - x).$$

a (30%) Solve this problem by first reduce it to one with homogeneous boundary conditions

b (10%) Analyze the behavior of u(x,t) as $t \to \infty$, find the equilibrium temperature distribution if it exists, explain why if the equilibrium temperature does not exit.